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In this final technical report, we outline a number of problems in robust and nonlinear control theory which we have studied using an operator theoretic methodology. In particular, we discuss an approach to the structured singular value and u-synthesis procedure in robust control which is based on a new type of interpolation method, We consider nonlinear extension of H-infinity optimization theory and investigate corresponding question in causality which arise in this area. Moreover, we discuss a saddle-point, game-theoretic approach to such problems. Further, we sketch the application of our "skew Toeplitz" methods to distributed (infinite dimensional) H-infinity control. We also consider and h-infinity of approach to sampled data systems based on a lifting method, as well as consider problem using h-infinity - H^2 combined suboptimal controller. Finally we list some of our results in image processing and computer vision as applied to visual tracking.

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Final Report for the AFOSR Contract AFOSR-90-0024 entitled "Operator Theoretic Methods in the Control of Distributed and Nonlinear Systems"

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1 Introduction

In our AFOSR work, we have carried out an extensive research program for the study of robust system control using methodologies from interpolation theory, dilation theory, and functional analysis. We have also become interested in image processing and computer vision, and their application to visual tracking problems.

First of all, we devoted a large portion of our research effort to nonlinear systems. This has led us to derive an *iterative commutant lifting theorem* which gives an explicit design procedure for nonlinear systems and captures the H^∞-control problem in the nonlinear framework [51], [52], [53]. We are of course interested in computer implementations of this work. In this area, we have defined a notion of *rationality* for nonlinear systems, and we have proven that the iterative commutant lifting procedure produces rational controllers (in this nonlinear sense) if we start from rational data. The procedure has already been applied to certain systems (connected with the National Aerospace Plane) with input saturations (in collaboration with colleagues at the Systems Research Center of Honeywell in Minneapolis). Moreover, these ideas are presently being applied to a disturbance attenuation problem in connection with the control of the ATB-1000 Testbed which models the gun turret of the Apache helicopter (in collaboration with control engineers at the Picatinny Arsenal in New Jersey) [24, 25]. This framework has also led to new directions in introducing notions of causality [54] in commutant lifting theory, and moreover has led to the possible formulation of a global nonlinear commutant lifting theorem as a saddle-point result. We are particularly interested in the treatment (and understanding the control limitations) of "hard" nonlinearities such as dead-zone, backlash, and saturation. These ideas will be explained below in our **Proposed Work**.

During the research period supported by AFOSR [illegible]J-0024, we also developed novel interpolation methods which are *not* norm-based. The[illegible]ve arisen out of our research into the multivariable gain margin problem as well as the more general *structured singular value*. We believe that this new type of interpolation theory should lead to several interesting directions in operator theory as well.

More precisely, our work on the multivariable gain margin problem, has led to a novel interpolation scheme which we call **spectral Nevanlinna-Pick interpolation** and the more general **structured interpolation** theory; see [19], [18], [20], and [23]. This involves matrix interpolation in which one bounds the spectral radius, and not the norm of the interpolants as in ordinary Nevanlinna-Pick theory. (Ordinary Nevanlinna-Pick is precisely the type of mathematical problem that arises in standard H^∞ synthesis.) We now have a theoretical procedure for solving this spectral interpolation problem both in its pure matricial and tangential formulations.

Our solution involves a generalization of the commutant lifting theorem, and a new object of interest in linear algebra and operator theory which we call the *T-spectral radius*. We are presently studying both the theoretical and practical ramifications of our solution, and we would like to develop software for the computation of the T-spectral radius. In our new research, we have been concentrating on the generalization of these results to the structured

singular value of Doyle and Safonov in order to try to develop an analytic μ-synthesis procedure [20]. Closely related to this work is a novel lifting technique which allows us to study the robust stability analysis of systems under various kinds of structured perturbations which we will consider below. (See also [13], [14], [16], and [86].) This lifting technique allows one to interpret the upper bound for the structured singular value μ defined in terms of certain scalings as a structured singular value on a larger space. Moreover, we will discuss below a new approach for the approximation of Riemann mappings (conformal equivalences from simply connected sub-domains of the complex plane to the unit disc) for the application of solving the gain-phase margin problem [66].

We have continued our research on the utilization operator theoretic methods in H^∞-optimization theory using *skew Toeplitz operators* (see [17] for the precise definition), which seem ideally suited for studying H^∞ design problems, especially for distributed (i.e., infinite dimensional) systems. These methods are quite natural in the control context since they allow one to do design just using the input/output operators. This has led to an explicit solution of the four block problem for large class of multivariable distributed systems. (This is the most general H^∞-optimization problem.) The procedure seems to be numerically robust as evidenced by its implementation at the Systems Research Center of Honeywell, and its application to a number of distributed plant models including a flexible beam [70], and an unstable delay system associated with the flight control of the X-29 [35]. A nice feature of this approach is that the complexity of the computations only depends on the the weighting matrices (modelling the disturbances) and not on the plant (which may be distributed). Since the weighting matrices are typically taken to be rational, this approach seems very efficient even in the finite dimensional case for plants with large state spaces. We have also been developing methods for the utilization of an operator due to Young [94] in order to simplify some of our computations in the *standard problem* for multivariable systems.

Recently there has been a large amount of work devoted to state space approaches to H^∞ optimization; see [3], [33], [58] and the references therein. In our AFOSR research, we have verified a formula which combines the state and frequency approaches to H^∞-optimal design in the one block (sensitivity minimization) case [71], [96]. We will also be interested in extending this to the more general two and four block frameworks. This type of formula should combine the advantages of both the elegant state space (Riccati equation) and frequency domain (input/output) approaches to H^∞ theory.

Next using the one-step dilation procedure, we have given a way of parametrizing the suboptimal controllers for such generalized interpolation problems [43]. This has led to a scheme for designing suboptimal finite dimensional controllers for distributed systems in the one block setting [77]. We would like to continue this direction for the general standard problem. as well as apply the techniques to a number of design problems. Some steps in this direction have already been taken in [35], [70], and [72].

Closely related to the above work has been the use of H^∞ techniques to study sampled-data control. See [11] and the references therein. Here one can use a certain operator-theoretic lifting method to describe a complete solution to the analysis problem of verifying that a given controller constrains the L^2-induced norm of the sampled-data system to be less than some

pre-specified level. (In fact, the lifting method is applicable to all norm based optimization problems. In discrete-time such a technique was employed in [64] to study a number of issues in robust control.)

We will also discuss our research in combined H^∞-H^2 suboptimal controllers ([36] and [38]) as well as sketch an approach for combining other norms based on techniques from *interpolation on Banach spaces*, and the connection of this methodology to a new approach for model reduction which we intend to explore in our new AFOSR contract.

We should note that our work in H^∞ control has also motivated a number of interpolation theoretic problems connected to a strong version of the Parrott theorem [50], and the resulting theory of minimal entropy extensions [37]. Moreover, there are a number of interesting operator theoretic problems connected to the singular spectrum of the four block operator which we will describe in this proposal, and which we intend to explore further. Finally, there are some deep operator theoretic problems related to the characterization of the optimal solutions in spectral interpolation theory [18].

In a different direction, we have been using curve evolution theory (and especially invariant nonlinear diffusion equations), for the development of new algorithms in image processing and computer vision. Curve evolution theory has recently become a major topic of research, and indeed has appeared in such diverse fields as differential geometry, parabolic equations theory, numerical analysis, computer vision, the viscosity solutions of Hamilton-Jacobi equations, and image processing. See [67, 68, 69, 81, 82, 83, 84, 85] and the references therein.

In particular, evolution equations which are geometric non-linear versions of the classical heat equation have received much attention, since these equations have both theoretical and practical importance. These ideas have been used also in shape-from-shading, image smoothing and enhancement, motion planning, shape segmentation, optical flow, and a continuous implementation of mathematical morphology.

A complete list of publications which acknowledges support of AFOSR-90-0024 is included in Section 7. We now sketch the work done by A. Tannenbaum and collaborators on this contract.

2 Nonlinear Robust Control

We have been pursuing several directions in order to derive nonlinear generalizations of the (linear) H^∞ theory in the weighted sensitivity minimization (one block), mixed sensitivity (two block), and even standard problem frameworks. One such direction is based on an **iterative commutant lifting theorem** [51], [52], [53], [41] which gives an explicit design procedure for nonlinear systems and captures the H^∞*-control* problem for a large class of nonlinear plants. We have also defined a notion of *rationality* for nonlinear systems, and we have proven that the iterative commutant lifting procedure produces rational controllers (in this nonlinear sense) if we start from rational data [51]. We have thus been able to write computer code for this procedure along the lines that was done for the four block problem using the theory of skew Toeplitz operators.

2.1 Causal Analytic Mappings

We consider analytic mappings on Hilbert space. For the precise definitions see [51], [52], [53], [39]. For simplicity, we will only consider SISO operators in what follows. The multivariable case works the same way [39]. We will find it convenient to employ the Fourier representation of elements of $H^2(D^n)$.

So we consider an analytic map ϕ with $\mathcal{G} = \mathcal{H} = H^2$ (the standard Hardy space on the disc D). Note that

$$H^2 \otimes \cdots \otimes H^2 = (H^2)^{\otimes n} \cong H^2(D^n)$$

where we map $1 \otimes \cdots \otimes z \otimes \cdots \otimes 1$ (z in the i-th place) to z_i, $i = 1, \cdots, n$. In the usual way, we say that ϕ *shift-invariant* (or *time-invariant*) if

$$\phi_n S^{\otimes n} = S\phi_n \quad \forall n \geq 1,$$

where $S : H^2 \to H^2$ denotes the canonical unilateral right shift. (Equivalently, this means that $S\phi = \phi \circ S$ on some open ball about the origin in which ϕ is defined.)

Now set

$$P_{(j)}^{(n)} := P_{(j)} \otimes \cdots \otimes P_{(j)} \;\; (n \text{ times}), \quad j \geq 1, \;\; n \geq 1,$$

where

$$P_{(j)} := I - S^j S^{*j}.$$

Then we say that ϕ is *causal* if

$$P_{(j)}\phi_n = P_{(j)}\phi_n P_{(j)}^{(n)}, \quad j \geq 1, \;\; n \geq 1.$$

For $\phi : H^2 \to H^2$ linear and time-invariant, it is easy to see that ϕ is causal. In the nonlinear setting however, time-invariance may not imply causality [53]. It is for this reason, that a causality constraint must be explicitly included for nonlinear H^∞ design.

2.2 Causal Optimization Problem

Because of the difficulties involving causality when one applies the classical commutant lifting theorem in the nonlinear framework, we will need to formulate a new linear causal optimization problem. Then we will indicate how to reduce the nonlinear generalization of the H^∞ sensitivity minimization problem to a series of such problems.

We let $S_{(n)}$ denote the unilateral shift on $H^2(D^n)$ given by multiplication by $(z_1 \cdots z_n)$. Since $H^2(D^n)$ will be fixed in the discussion we will let $S := S_{(n)}$. In what follows, U will denote the unilateral shift on H^2 given by multiplication by z, and $\Theta \in H^\infty$ will be an inner function. Finally $W : H^2(D^n) \to H^2$ will denote a causal, time-invariant bounded linear operator.

We can now state the *causal H^∞-optimization problem* (COP): Find

$$\sigma := \inf\{\|W - \Theta Q\| : Q : H^2(D^n) \to H^2, \; Q \text{ causal, time-invariant}\}. \tag{1}$$

Moreover, we want to compute an optimal, causal, time-invariant Q_{opt} such that

$$\sigma = \|W - \Theta Q_{opt}\|. \tag{2}$$

If we drop the causality constraint, the solution to problem (1) is provided by the classical commutant lifting theorem [89]. With the causality constraint, the solution to (COP) is abstractly provided by a causal commutant lifting theorem [54], [42], [40].

Our constructions are based on a *reduction theorem* [39], which we will briefly sketch below. In order to formulate this result, we will first discuss the Fourier representation.

Fourier Representation

We need to define all the relevant spaces. Denote by

$$\ell^2(H^2) := \bigoplus_{i=1}^{\infty} H^2,$$

the Hilbert space of all column vectors

$$f(z) = [f_1(z), f_2(z), \ldots, f_n(z), \ldots]', \tag{3}$$

($'$ stands for tranpose) such that

$$\|f\|^2 := \sum_{i=1}^{\infty} \|f_i\|^2, \tag{4}$$

is finite. ($\| \ \|$ is our generic symbol for a Hilbert space norm (2-norm) as well as the induced operator norm. So for example in (4), it stands for the usual norm on H^2 as well as the associated norm on $\ell^2(H^2)$.) Thus $\ell^2(H^2)$ is a vector-valued Hardy space. Indeed, if $f(z)$ is given by (3), then we may write

$$f(z) = \sum_{m=0}^{\infty} a_m z^m, \tag{5}$$

where each a_m is an infinite column vector with components in $\mathbf{C}$, and

$$a_m = \frac{1}{m!}[f_1^{(m)}(0), \ldots, f_j^{(m)}(0), \ldots]'.$$

Clearly,

$$\|f\|^2 = \sum_{m=0}^{\infty} \|a_m\|^2.$$

Conversely, if $f(z) \in \ell^2(H^2)$ is given in the form (5) for

$$a_m = [a_{m1}, \ldots, a_{mj}, \ldots]',$$

then $f(z)$ can be written in the form (3), i.e.,

$$f(z) = [f_1(z), \ldots, f_j(z), \ldots]',$$

where

$$f_j(z) = \sum_{m=0}^{\infty} a_{mj} z^m.$$

In what follows, we will either use representation (3) or (5). The context should always make the meaning clear.

Next we let $S_\Phi : \ell^2(H^2) \to \ell^2(H^2)$ denote the unilateral shift defined by multiplication by z. Then the *Fourier representation* is given by the (linear, bounded) operator

$$\Phi := \Phi_n : H^2(D^n) \to \ell^2(H^2),$$

which is defined by

$$\begin{aligned} f(z) &:= \Phi(F(z_1, z_2, \ldots, z_n)) \\ &:= \sum_{m=0}^{\infty} z^m \begin{bmatrix} F_{m,m,\ldots,m} \\ F_{m+1,m,\ldots,m} \\ \vdots \\ F_{m,m,\ldots,m+1} \\ F_{m+2,m,\ldots,m} \\ \vdots \end{bmatrix}, \end{aligned} \tag{6}$$

where

$$F(z_1, \ldots, z_n) = \sum_{i_1,\ldots,i_n \geq 0} F_{i_1,\ldots,i_n} z_1^{i_1} \cdots z_n^{i_n}.$$

Note that we are taking $f(z)$ in the form (5) in the above representation. Moreover, note that

$$H^2(D^n) = \{F(z_1, \ldots, z_n) = \sum_{i_1,\ldots,i_n \geq 0} F_{i_1,\ldots,i_n} z_1^{i_1} \cdots z_n^{i_n} : \sum_{i_1,\ldots,i_n \geq 0} \|F_{i_1,\ldots,i_n}\|^2 < \infty\}.$$

We can also write

$$f(z) = [f_{0,\ldots,0}(z), f_{1,\ldots,0}(z), f_{0,\ldots,1}(z), f_{2,\ldots,0}(z), \ldots]', \tag{7}$$

where

$$f_{i_1,\ldots,i_n}(z) := \sum_{m=0}^{\infty} F_{i_1+m,\ldots,i_n+m} z^m. \tag{8}$$

Notice, we have chosen the indexing of the $f_{i_1,\ldots,i_n}$ in such a way that the indices run over the set

$$I_n := \{(i_1, \ldots, i_n) : i_1, \ldots, i_n \geq 0, \ \min\{i_1, \ldots, i_n\} = 0\}. \tag{9}$$

Next, it is easy to prove that $\Phi : H^2(D^n) \to \ell^2(H^2)$ is unitary, and that

$$\Phi S = S_\Phi \Phi. \tag{10}$$

By (10), we see that if $W : H^2(D^n) \to H^2$ is such that $WS = UW$, then the operator $W\Phi^* : \ell^2(H^2) \to H^2$ satisfies

$$(W\Phi^*)S_\Phi = WS\Phi^* = U(W\Phi^*),$$

that is, $W\Phi^*$ intertwines the shifts S_Φ and U. Consequently, it is standard that $W\Phi^*$ is represented by a row vector

$$[W_{0,\ldots,0}(z), W_{1,\ldots,0}(z), \ldots, W_{0,\ldots,1}(z), W_{2,\ldots,0}(z), \ldots]. \tag{11}$$

We will write that

$$W\Phi^* \cong [W_{0,\ldots,0}(z), W_{1,\ldots,0}(z), \ldots, W_{0,\ldots,1}(z), W_{2,\ldots,0}(z), \ldots]. \tag{12}$$

One may show that in fact

$$W_{i_1,\ldots,i_n}(z) = W(z_1^{i_1} \cdots z_n^{i_n}), \quad (i_1, \ldots, i_n) \in I_n. \tag{13}$$

The above discussion used only the time-invariance for W. In the next proposition, we will write down an explicit expression for the row vector of (13) associated with $W\Phi^*$ in case W is causal.

Proposition 1 *Let $W : H^2(D^n) \to H^2$ be time-invariant. Then W is causal if and only if*

$$W_{i_1,\ldots,i_n}(z) = z^{\max\{i_1,\ldots,i_n\}} W^c_{i_1,\ldots,i_n}(z), \ \ \forall (i_1, \ldots, i_n) \in I_n$$

where $W^c_{i_1,\ldots,i_n}(z) \in H^\infty$.

By the above discussion (in particular, Proposition 1), we see that for W, Θ as in the (COP) problem (1), we have

$$\begin{aligned}
\sigma &= \inf\{\|W - \Theta Q\| : QS = UQ, Q \text{ causal, time-invariant}\} \\
&= \inf\{\|W\Phi^* - \Theta Q\Phi^*\| : (Q\Phi^*)S_\Phi = U(Q\Phi^*), Q \text{ causal, time-invariant}\} \\
&= \inf\{\|W_1 - \Theta Q_1\| : W_1, Q_1 : \ell^2(H^2) \to H^2,\ W_1 = W\Phi^*, \\
&\quad Q_1 \cong [q_{0,\ldots,0}(z), zq_{1,\ldots,0}(z), \ldots, zq_{0,\ldots,1}(z), z^2 q_{2,\ldots,0}(z), \ldots]\}.
\end{aligned}$$

From now on (unless explicitly stated otherwise), we will just work with the operators $W_1, Q_1 : \ell^2(H^2) \to H^2$. Essentially, via the unitary equivalence Φ, we are identifying the spaces $H^2(D^n)$ and $\ell^2(H^2)$. In particular, we identify W with $W_1 = W\Phi^*$, and Q with $Q_1 = Q\Phi^*$. For simplicity of notation, we will denote

$$W = W_1, \quad Q = Q_1.$$

The context should always make the meaning clear.

We now translate the notions of causality and time-invariance for an operator $W : \ell^2(H^2) \to H^2$. We will say that W is *time-invariant* if $WS_\Phi = UW$, that is,

$$W \cong [W_{0,\ldots,0}(z), W_{1,\ldots,0}(z), \ldots, W_{0,\ldots,1}(z), W_{2,\ldots,0}(z), \ldots].$$

Moreover, we say that W is *causal* if the operator $W\Phi : H^2(D^n) \to H^2$ is causal, which means (see Proposition 1) that

$$W \cong [W^c_{0,\ldots,0}(z), zW^c_{1,\ldots,0}(z), \ldots, zW^c_{0,\ldots,1}(z), z^2W^c_{2,\ldots,0}(z), \ldots],$$

for some

$$\{W^c_{i_1,\ldots,i_n}(z) \in H^\infty : (i_1, \ldots, i_n) \in I_n\}.$$

Motivated by the above discussion, for $W : \ell^2(H^2) \to H^2$ time-invariant and causal, we introduce the operator

$$\begin{aligned} W_c &\cong [W^c_{0,\ldots,0}(z), W^c_{1,\ldots,0}(z), \ldots, W^c_{0,\ldots,1}(z), W^c_{2,\ldots,0}(z), \ldots] \\ &= [W_{0,\ldots,0}(z), W_{1,\ldots,0}(z)/z, \ldots, W_{0,\ldots,1}(z)/z, W_{2,\ldots,0}(z)/z^2, \ldots]. \end{aligned} \tag{14}$$

Recapping, in order to solve the (COP) problem (1), we can equivalently solve the following problem: Given $W : \ell^2(H^2) \to H^2$ time-invariant and causal as above, find

$$\sigma = \inf\{\|W - \Theta Q\| : QS_\Phi = UQ,\ Q \text{ causal}\}. \tag{15}$$

Thus we have to solve the optimization problem (COP) on the Fourier transformed operators. But this can be easily accomplished via the following key result that we proved in [39].

Theorem 1 (Reduction Theorem) *Notation as above. Then*

$$\sigma = \inf\{\|W - \Theta Q\| : QS = UQ,\ Q \textit{ causal}\} \tag{16}$$

$$= \inf\{\|[W_{0,\ldots,0}(z) - \Theta q_{0,\ldots,0}(z), z(W_{1,0,\ldots,0}(z) - \Theta q_{1,0,\ldots,0}(z)), \ldots]\| : \tag{17}$$

$$[q_{0,\ldots,0}(z), \ldots, q_{2,\ldots,0}(z), \ldots] \in \mathcal{L}(\ell^2(H^2), H^2)\}$$

$$= \inf\{\|W_c - \Theta Q\| : QS = UQ\}. \tag{18}$$

(Note in (17) the norm is the operator norm in $\mathcal{L}(\ell^2(H^2), H^2)$. In general, for Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$, $\mathcal{L}(\mathcal{H}, \mathcal{K})$ denotes the space of bounded linear operators from $\mathcal{H}$ to $\mathcal{K}$.)

Algorithm for Computation of σ

We would like to summarize the above discussion with a high-level algorithm for the computation of the optimal causal performance σ, and corresponding causal optimal interpolant Q_{opt} in (1) and (2).

First of all using the notation of the Reduction Theorem, let us denote

$$\sigma_o := \inf\{\|W_c - \Theta Q\| : QS = UQ\}. \tag{19}$$

(See equation (18).) Then the Reduction Theorem guarantees that

$$\sigma = \sigma_o.$$

This means that a causal optimization problem can be reduced to a classical generalized interpolation problem in H^∞.

We can summarize the procedure as follows:

(i) Let W, Θ be as in (1). (Thus $W : H^2(D^n) \to H^2$ here.) We compute $W(z_1^{i_1} \cdots z_n^{i_n})$ where $(i_1, \ldots, i_n) \in I_n$. We get that

$$W\Phi^* \cong [W_{0,\ldots,0}(z), W_{1,\ldots,0}(z), \ldots, W_{0,\ldots,1}(z), W_{2,\ldots,0}(z), \ldots],$$

and then as above, we can obtain the row matrix

$$[W_{0,\ldots,0}(z), W^c_{1,\ldots,0}(z), \ldots, W^c_{0,\ldots,1}(z), W^c_{2,\ldots,0}(z), \ldots].$$

(ii) The latter row matrix represents an operator $W_c : \ell^2(H^2) \to H^2$. Let $\Pi : H^2 \to H^2 \ominus \Theta H^2$ denote orthogonal projection. Using skew Toeplitz theory, we can compute the norm of the operator

$$\Lambda(W, \Theta) := \Pi W_c. \tag{20}$$

The norm of this operator is σ, the optimal causal performance.

(iii) Using the classical commutant lifting theorem and skew Toeplitz theory, we can compute the optimal dilation $B_c : \ell^2(H^2) \to H^2$ of $\Lambda(W, \Theta)$. Recall this means that

$$B_c S_\Phi = U B_c, \quad \Pi B_c = \Lambda(W, \Theta), \quad \|B_c\| = \|\Lambda(W, \Theta)\| = \sigma.$$

We can then write

$$B_c = W_c - \Theta Q_{opt,c}.$$

Then from (14), we can find the optimal causal dilation

$$B = W\Phi^* - \Theta Q_{opt}\Phi^*.$$

Note that B and B_c are related as in (14), and similarly for $Q_{opt,c}$ and $Q_{opt}\Phi^*$. $Q_{opt} : H^2(D^n) \to H^2$ is the optimal causal interpolant, i.e.,

$$\sigma = \|W - \Theta Q_{opt}\|.$$

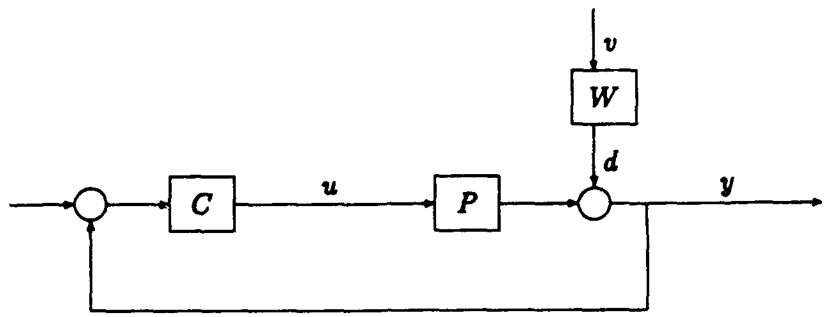

Figure 1: Sensitivity Minimization Configuration

Now in analogy to our previous work, we can base an **causal iterative design procedure** on this chain of ideas.

Here is how such a procedure would look. Let us call an analytic input/output operator $\phi : H^2 \to H^2$ *admissible* if is is causal, time-invariant, and $\phi(0) = 0$. Denote the set of admissible operators by $\mathcal{C}_a$. In what follows below, we assume $P, W \in \mathcal{C}_a$, and that W admits an admissible inverse.

Referring to Figure 1, we consider the problem of finding

$$\mu_\delta := \inf_C \sup_{\|v\| \leq \delta} \|[(I + P \circ C)^{-1} \circ W]v\|, \tag{21}$$

where we take the infimum over all stabilizing controllers. (In what follows, we let $\|\ \|$ denote the 2-norm $\|\ \|_2$ on H^2 as well as the associated operator norm. The context will make the meaning clear.) Thus we are looking at a worst case disturbance attenuation problem where the energy of the signals v is required to be bounded by some pre-specified level δ. (In the linear case of course since everything scales, we can always without loss of generality take $\delta = 1$. For nonlinear systems, we must specify the energy bound *a priori*.) Then one sees that (21) is equivalent to the problem of finding the problem of finding

$$\mu_\delta = \inf_{q \in \mathcal{C}_a} \sup_{\|v\| \leq \delta} \|(W - P \circ q)v\|. \tag{22}$$

The iterated causal commutant lifting procedure gives an approach for approximating a solution to such a problem. Briefly, the idea is that we write

$$\begin{aligned} W &= W_1 + W_2 + \cdots, \\ P &= P_1 + P_2 + \cdots, \\ q &= q_1 + q_2 + \cdots, \end{aligned}$$

where W_j, P_j, q_j are homogeneous polynomials of degree j. Notice that

$$\mu_\delta = \delta \inf_{q_1 \in H^\infty} \|W_1 - P_1 q_1\| + O(\delta^2), \tag{23}$$

where the latter norm is the operator norm (i.e., H^∞ norm). From the classical commutant lifting theorem we can find an optimal (linear, causal, time-invariant) $q_{1,opt} \in H^\infty$ such that

$$\mu_\delta = \delta \|W_1 - P_1 q_{1,opt}\| + O(\delta^2). \tag{24}$$

Now the iterative procedure gives a way of finding higher order corrections to this linearization. Let us illustrate this now with the second order correction. Indeed, having fixed the linear part $q_{1,opt}$ of q in (22), we note that

$$W(v) - P(q(v)) - (W_1 - P_1 q_{1,opt})(v) =$$

$$W_2(v) - P_2(q_{1,opt}(v)) - P_1 q_2(v) + \text{higher order terms.}$$

Regarding W_2, P_2, q_2 as linear operators on $H^2 \otimes H^2 \cong H^2(D^2, \mathbf{C})$ as above, we see that

$$\sup_{\|v\| \le \delta} \|(W - P \circ q)(v) - (W_1 - P_1 q_{1,opt})v\| \le \delta^2 \|\hat{W}_2 - P_1 q_2\| + O(\delta^3),$$

where the "weight" $\hat{W}_2$ is given by

$$\hat{W}_2 := W_2 - P_2(q_{1,opt} \otimes q_{1,opt}).$$

The point of the iterative causal commutant lifting procedure is to allow us to construct an optimal admissible $q_{2,opt}$, and so on.

In short, instead of simply designing a linear compensator for a linearization of the given nonlinear system, this methodology allows one to explicitly take into account the higher order terms of the nonlinear plant, and therefore increase the ball of operation for the nonlinear controller. Moreover, if the linear part of the plant is rational, our iterative procedure may be reduced to a series of finite dimensional matrix computations. (See [39, 51] for discussions of rationality in the nonlinear framework.)

Finally, in a number of interesting cases, the above methodology can be extended to a nonlinear extension of the mixed sensitivity problem; see [34]. In fact, one can show that for linear weighting filters and admissible plant, the nonlinear mixed sensitivity problem may be reduced to a standard linear two block problem, followed by a one block nonlinear design, which may be solved using the above iterative methods.

Recently the above procedure was extended to the *general standard problem for nonlinear systems*; see [41]. We should note that we have concentrated on time-invariant systems in our above description. In fact, there is a much more general causal commutant lifting methodology [54, 42, 40] that can be extended in principle to *time-varying* systems. This is a very important problem area. Moreover, there are interesting problems concerning the equality of certain invariants which arise the the general causal commutant lifting framework [54] which we intend to consider in our upcoming AFOSR contract.

2.3 Saddle-Point Approach to Nonlinear Optimization

We have discussed above an iterative commutant lifting approach to nonlinear system design. The iterative communtant lifting technique is basically a local analytic method for nonlinear system synthesis. We have also been exploring a very different approach applicable to certain systems with saturations based on an interpretation of the classical commutant lifting theorem as a saddle-point result [55]. This motivates us to formulate a nonlinear commutant lifting result in such a saddle-point, game-theoretic framework.

A related approach to nonlinear design has already been independently employed in the novel and important papers of Ball-Helton [8], [9]. See also the recent nice work in [59, 60].

In our research, instead of considering general nonlinear systems we limit ourselves to the concrete (but certainly interesting case) of linear systems with input saturations. Such systems occur, of course, all the time in "nature." We should add that a similar approach should be valid for many of the hard, memoryless, noninvertible nonlinearities which appear in control.

Mathematically, the case of the saturation is very interesting since in a certain sense it is a nonlinear analogue of an inner (non-minimum phase) element, whose "spectrum" seems to be spread throughout the unit circle. Thus the problem of sensitivity minimization for such elements (that is, a form of weighted inversion) is particularly challenging, and will be a key topic to be studied in our upcoming research program. Again, preliminary results along these lines have been worked out in [55]. What needs to be developed still is a commutant lifting type result valid on convex spaces. This we plan to do in our new AFOSR contract.

3 Structured Singular Values and μ-Synthesis

We would like now to discuss our work on the structured singular value and μ-synthesis using a novel approach which we developed called *structured interpolation theory*. The structured singular value was introduced into robust control by John Doyle and Michael Safonov ([30], [31], [79]) to handle problems involving structured perturbations which includes both H^∞ and the multivariable gain margin as special cases.

Our starting point is the fact that the problem of internal stabilization of a given LTI multivariable plant can be reduced to one of interpolation. Thus the natural measure of robust performance in this framework is given by the minimization of the structured singular values over all possible interpolants. Hence, one needs a structured Nevanlinna-Pick type result which will generalize both the classical Nevanlinna-Pick theorem (relative to the H^∞-norm), as well as the spectral Nevanlinna-Pick theorem (relative to the spectral radius). Such a structured Nevanlinna-Pick theorem can be deduced as a consequence of a general structured commutant lifting theorem as we will indicate below. This approach has been developed in our AFOSR sponsored work [18, 19, 15, 16, 23, 12].

We will discuss some of these issues now, and explain how these results could serve as a basis of performing the μ-synthesis procedure in robust feedback control in a rigorous, analytical manner.

3.1 Structured Singular Values and Dilations

We would like to formally introduce the structured singular value now, and give some of its basic properties. We base this discussion on [20]. Instead of working over diagonal sets of matrices as in [30], we can more generally work over an arbitrary finite dimensional C^*-algebra.

Let $\mathcal{E}$ be a complex finite-dimensional Hilbert space, and $\Delta \subset \mathcal{L}(\mathcal{E})$ (the space of bounded linear operators on $\mathcal{E}$), a C^*-algebra. For $A \in \mathcal{L}(\mathcal{E})$, $A \neq 0$, we define the ***structured singular value***

$$\mu_\Delta(A) := [\inf\{\|X\| : X \in \Delta,\ -1 \in \sigma(AX)\}]^{-1}.$$

Moreover, we set

$$\widehat{\mu}_\Delta(A) := \inf\{\|XAX^{-1}\| : X \in \Delta'\},$$

where Δ' is the commutator of Δ. Note that for $\Delta = \mathcal{L}(\mathcal{E})$, $\mu_\Delta(A) = \|A\|$, while for $\Delta = \mathbf{C}I_\mathcal{E}$, $\mu_\Delta(A) = \|A\|_{sp}$ (the spectral radius of A).

We now summarize some of the elementary properties of μ_Δ based on Doyle [30]. The relevant properties are:

(i) $\mu_\Delta(A) = \sup\{\|AX\|_{sp} : X \in \Delta,\ \|X\| \leq 1\}$.

(ii) μ_Δ is continuous.

(iii) $\mu_\Delta(A) \leq \widehat{\mu}_\Delta(A)$.

For certain diagonal algebras of matrices, it is argued in [30], [79], and [80] that the structured singular value μ_Δ is the natural object of study in robust control. Unfortunately the structured singular value is difficult to compute, so in practice it is $\widehat{\mu}_\Delta$ which is actually used for control problems. It is therefore of interest to know when these two objects are equal.

In [30], Doyle has shown that in fact $\mu_\Delta = \widehat{\mu}_\Delta$ when the relevant diagonal algebra has three or fewer blocks. In [20], we give a very different proof of this fact based on the following result which we believe has independent interest. Define the operator $M_A \in \mathcal{L}(\mathcal{E})$ by $M_A := AX$. Notice that $\mathcal{L}(\mathcal{E})$ may be given a Hilbert space structure with respect to

$$\langle T_1, T_2 \rangle := Tr(T_2^* T_1),$$

where Tr denotes the trace. Define

$$\tilde{\mu}_\Delta(A) := \mu_{\tilde{\Delta}}(M_A)$$

where

$$\tilde{\Delta} := \{M_X : X \in \Delta'\}'.$$

We now have (see [20] for the proof):

Theorem 2

$$\widehat{\mu}_\Delta(A) := \bar{\mu}_\Delta(A).$$

Note that Theorem 8 implies that $\widehat{\mu}_\Delta$ can be regarded as a structured singular value on a bigger space. From the theorem and property (ii) above, we can immediately infer the key fact that $\widehat{\mu}_\Delta$ is continuous. This result is also strongly connected to some recent work on robust stability with respect to time-varying perturbations [86]. In fact, we are investigating the extension of this lifting technique to operators on infinite dimensional Hilbert spaces with applications to the robust stability analysis of systems under various kinds of structured perturbations [13]. In particular, we have considered the following two cases [13, 14, 21]:

(i) The algebra of constant diagonal scales Δ', with the operator A taken to be analytic Toeplitz. (This is the case analyzed in [86].) Note that by "analytic Toeplitz operator" A, we mean that A is given by an $n \times n$ matrix with entries H^∞ functions, which we regard as acting as a multiplication operator on $H^2(\mathbf{C}^n)$. Thus A defines a stable LTI system. We also want to consider general time-varying linear input-output operators with this constant diagonal algebra of scalings as well.

(ii) Again we take A to be analytic Toeplitz, but now we want to consider Δ' to be the algebra of diagonal analytic Toeplitz operators. This is the type of problem considered in μ-synthesis.

We should note that this work has also led to a new *relative Toeplitz-Hausdorff theorem*.

We will now outline a structured analogue of the commutant lifting theorem [20]. This will be applied to the structured version of classical matricial Nevanlinna-Pick interpolation below.

Set

$$T := S(m) \otimes I_{\mathcal{E}}$$

where m is a finite Blaschke product, $S(m)$ is the compressed shift, and $\mathcal{E}$ is a finite dimensional complex Hilbert space. Fix $\Delta \subset \mathcal{L}(\mathcal{E})$, a C^*-algebra. Define

$$I_{H^2} \otimes \Delta := \{I_{H^2} \otimes X : X \in \Delta\},$$

$$I_{H^2} \otimes \Delta' := \{I_{H^2} \otimes X : X \in \Delta'\}.$$

Notice that $\mathcal{H} := H^2(\mathcal{E}) \ominus mH^2(\mathcal{E})$ reduces both $I_{H^2} \otimes \Delta$ and $I_{H^2} \otimes \Delta'$. Now define for $A \in \{T\}'$ (the commutant of T),

$$\rho_T^\Delta(A) := \inf\{\|XAX^{-1}\| : X \text{ invertible, } X \in \{T\}',\ X \in (I_{H^2} \otimes \Delta|\mathcal{E})'\}.$$

Let U be the isometric dilation of T on $H^2(\mathcal{E})$ (so that U is defined by multiplication by z). For $B \in \{U\}'$, define

$$\rho_U^\Delta(B) := \inf\{\|YBY^{-1}\| : Y \text{ invertible, } Y \in \{U\}',\ Y \in (I_{H^2} \otimes \Delta)'\}.$$

We can now state the following result from [20]:

Theorem 3 (Structured Commutant Lifting Theorem) *Notation as above. Then for* $A \in \{T\}'$,

$$\rho_T^{\Delta}(A) = \inf\{\rho_U^{\Delta}(B) : B \textit{ is a commuting dilation of } A\}.$$

Actually, in the finite dimensional case of interest to us, we can show that

$$\rho_T^{\Delta}(A) = \inf_B \sup_{z \in D} \widehat{\mu}_{\Delta}(B(z))$$

where $B \in \{U\}'$ is a commuting *rational* dilation of A.

3.2 Structured Nevanlinna-Pick Interpolation

We now apply the above theory to a structured version of the Nevanlinna-Pick interpolation problem for matrix-valued interpolants.

Let $\mathcal{E}$ be a finite dimensional Hilbert space, let $z_1, \ldots, z_n \in D$ be distinct, and let $F_1, \ldots, F_n \in \mathcal{L}(\mathcal{E})$. We begin with the classical matricial Nevanlinna-Pick problem in which we want necessary and sufficient conditions for the existence of an analytic function $F : D \to \mathcal{L}(\mathcal{E})$ with $\|F\|_\infty < 1$ such that

$$F(z_j) = F_j \tag{25}$$

for $j = 1, \ldots, n$. Define

$$m(z) := \prod_{j=1}^{n} \frac{z - z_j}{1 - \overline{z}_j z},$$

$$f_j := \left(\prod_{k \neq j} \frac{z - z_k}{1 - \overline{z}_k z} \right) \frac{1}{1 - \overline{z}_j z},$$

and

$$\mathcal{H} := H^2(\mathcal{E}) \ominus m H^2(\mathcal{E}).$$

We set $T := S(m) \otimes I_{\mathcal{E}}$, and note that

$$\mathcal{H} = f_1 \otimes \mathcal{E} + f_2 \otimes \mathcal{E} + \cdots + f_n \otimes \mathcal{E}.$$

This sum is direct but not orthogonal.

For the given interpolation data above, we define $A : \mathcal{H} \to \mathcal{H}$ by linearity and by

$$A(f_j \otimes \xi) := f_j \otimes F_j \xi$$

for all $\xi \in \mathcal{E}$, $j = 1, 2, \ldots, n$. Note that

$$T(f_j \otimes \xi) = z_j f_j \otimes \xi$$

for all $\xi \in \mathcal{E}$, $j = 1, 2, \ldots, n$. Thus

$$A \in \{T\}'.$$

It is easy to show that F satisfies the interpolation conditions

$$F(z_j) = F_j, \quad \forall j = 1, \ldots, n \tag{26}$$

if and only if

$$P_{\mathcal{H}} M_F = A P_{\mathcal{H}},$$

where $M_F : H^2(\mathcal{E}) \to H^2(\mathcal{E})$ denotes the multiplication operator associated to F. Then the classical commutant lifting theorem implies that there exists an F satisfying (26) and $\|F\|_\infty < 1$ if and only if $\|A\| < 1$. We will now show that the structured Nevanlinna-Pick problem can be given a similar solution, based on the structured commutant lifting theorem.

More precisely, define

$$\mathcal{I} := \{F : D \to \mathcal{L}(\mathcal{E}) : F \text{ is rational, bounded in } D,\ F(z_j) = F_j\}.$$

We are interested in finding

$$\mu(\mathcal{I}) := \inf\{\sup_{z \in D} \mu_\Delta(F(z)) : F \in \mathcal{I}\}$$

$$\hat{\mu}(\mathcal{I}) := \inf\{\sup_{z \in D} \hat{\mu}_\Delta(F(z)) : F \in \mathcal{I}\}.$$

We denote the operator A associated with the matrices $F_1, \ldots, F_n$ by $A(F_1, \ldots, F_n)$. Set

$$\rho_T^\Delta(A) = \\ \inf\{\|A(M_1 F_1 M_1^{-1}, \ldots, M_n F_n M_n^{-1})\| \ : \ M_j \in \Delta',\ 1 \le j \le n\}$$

(where the M_j are invertible) and

$$\rho_{T,\Delta}(A) = \\ \sup\{\rho_T(A(F_1 X_1, \ldots, F_n X_n)) : \|A(X_1, \ldots X_n)\| \le 1,\ X_1, \ldots, X_n \in \Delta\},$$

where as above, for an operator $Q \in \{T\}'$ we let

$$\rho_T(Q) = \inf\{\|XQX^{-1}\| : X \text{ is invertible and } X \in \{T\}'\ \}.$$

It is easy to show that

$$\rho_T^\Delta(A) \ge \rho_{T,\Delta}(A).$$

We can now state [20]:

Theorem 4 (Structured Nevanlinna-Pick) *Notation as above. Then*

$$\hat{\mu}(\mathcal{I}) = \rho_T^\Delta(A).$$

Theorem 5 *Notation as above. Then*

$$\mu(\mathcal{I}) \geq \rho_{\mathcal{T},\Delta}(A).$$

Next in many control problems one is interested in a variant of Nevanlinna-Pick interpolation, called *tangential Nevanlinna-Pick interpolation.* In the tangential problem, we are given $z_1, \ldots, z_n \in D$ (which we take to be distinct for simplicity), $u_1, \ldots, u_n \in \mathbf{C}^N$ non-zero vectors, and $v_1, \ldots, v_n \in \mathbf{C}^N$ arbitrary vectors. We are then interested in those bounded analytic functions $F : D \to \mathcal{L}(\mathbf{C}^N)$ which satisfy the interpolation conditions

$$F(z_j)u_j = v_j, \quad j = 1, \ldots, n. \tag{27}$$

In the classical Nevanlinna-Pick problem, we ask for conditions guaranteeing the existence of an interpolating function F with

$$\|F\|_\infty := \sup\{\|F(z)\| : z \in D\} < 1.$$

In [15], we study the spectral version in which we require

$$\|F\|_{sp} := \sup\{\|F(z)\|_{sp} : z \in D\} < 1.$$

Define the *tangential Nevanlinna-Pick matrix* as follows (for $\rho > 0$):

$$\mathcal{N}(z_1, \ldots, z_n; u_1, v_1; \ldots; u_n, v_n; \rho) := \left[\frac{\langle \rho v_j, \rho v_k\rangle - \langle u_j, v_k\rangle}{1 - \bar{z}_j z_k}\right]_{1 \leq j,k \leq n}.$$

The following result may be deduced as a corollary of a *tangential spectral commutant lifting theorem* [15]:

Theorem 6 (Tangential Nevanlinna-Pick Theorem) *There exists $F \in H^\infty(\mathbf{C}^N)$ with $\|F\|_{sp} < \rho$ satisfying the interpolation conditions (27) if and only if there exist $X_j \in B(\mathbf{C}^N)$ $(1 \leq j \leq n)$ invertible such that*

$$\mathcal{N}(z_1, \ldots, z_n; X_1 u_1, X_1 v_1; \ldots; X_n u_n, X_n v_n; \rho) > 0.$$

Work is already in progress on the *two-sided* and *bitangential* spectral and structured analogues of the classical results [12]. These results are precisely what are needed for a completely rigorous μ–synthesis procedure in robust control.

3.3 Conformal Mappings

We would like to briefly discuss some work that we also did on the gain-phase margin problem that involves some novel approximation methods of Riemann mappings (conformal equivalences from a simply connected proper subset of C to the unit disc) due to Marshall-Morrow (see [27] and the references therein).

The successful solution of the gain and phase margin problems (see [90], [66], [32]) was based on the fact that one could explicitly construct a conformal equivalence from a simply connected subdomain of C (which was associated to the uncertainty model) to the disc. Nevanlinna-Pick interpolation then took care of the rest. In [66], we considered a combination of both the classical gain and phase margins in order to arrive at a better measure of robust stability which we called the *gain-phase margin*. Unfortunately, in this case, there is no explicitly computable conformal equivalence between the associated domain of uncertainty and the disc. The algorithm of Marshall-Morrow however gives a fast way of approximating the given equivalence, and has led to a reliable approximate solution of gain-phase margin in [27]. We are now applying this algorithm to the type of general robust synthesis question as considered by Sideris and Safonov in [87].

3.4 Problems in Classical Interpolation Theory

We would like to mention a direction in our research which involves some interesting problems in classical interpolation theory, in particular, the minimal entropy solutions for a number of interpolation problems [50], [37]. We have found that in studying the spectral properties of the four block operator and its relation to interpolation theory, we have been led to a strong version of the classical Parrott's theorem. Parrott's theorem is one of the key matrix extension results and has found numerous uses in control theory as well as signal processing. Our strengthened version in a certain sense picks out an extension which is "opposite" to the famous maximal entropy or central solution to such extension problems. This solution has a natural physical interpretation in the waves through multi-layered media context. From the more theoretical side, this result can be used in generalizing some beautiful results of Adamjan-Arov-Krein on the connection of the singular values of the Hankel operator to optimal interpolation by functions with a presribed number of poles on the unit disc to the four block operator of H^∞ control [46].

Further, based on the strong Parrott theorem we have proven a strong version of the commutant lifting theorem [37] which leads to an explicit parametrization of minimal entropy solutions in dilation theory. We would like to apply this to the Nevanlinna-Pick and Caratheodory interpolation problems (and even to the general standard H^∞ problem), in order to explore the system theoretic consequences of this class of dilations.

4 H^∞ Optimization of Distributed Systems

In [73], [74] we have considered the mixed sensitivity H^∞-optimization problem for distributed plants with a finite number of unstable poles.

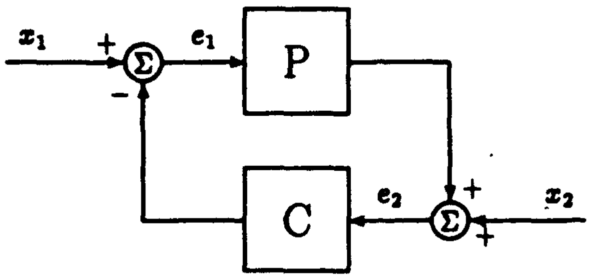

Figure 2: Standard Feedback Configuration

As in the standard operator approach to such problems, the computation of the optimal performance and corresponding optimal controller is reduced to a finite dimensional matrix problem. In the stable case, the size of the matrix only depends on the McMillan degree of the weighting filters. In the case of unstable plants, the size of the corresponding matrix depends on the number of right half plane poles of the plant as well. The dimension of this matrix can be computed *a priori*. The key mathematical fact that we use is that the skew Toeplitz operators which we obtain in the unstable case are finite rank perturbations of the classical skew Toeplitz operators obtained from compressions of rational functions.

In our AFOSR research, we have used these techniques to synthesize controllers for several types of flexible structures [70], and an unstable delay system derived from the flight control of the X-29 [35].

4.1 Two Block Problems With Unstable Plant

In this section, we will show that several two block H^∞-minimization problems reduce to the computation of the norm of a certain skew Toeplitz operator, and indicate how this norm may be computed. We begin with some notation. The Hardy spaces H^2 and H^∞ are defined on the unit disc as above. We denote

$$RH^\infty := \{\text{rational functions in } H^\infty\}.$$

We consider the feedback configuration of Figure 2 with

$$P = \frac{G_n}{G_d}$$

and $G_n \in H^\infty$, $G_d \in RH^\infty$.

We assume that (i) $G_n = m_n G_{no}$, where $m_n \in H^\infty$ is inner (arbitrary) and $G_{no} \in H^\infty$ is outer, and (ii) G_n, G_d have no common zeros in the closed unit disc. We also write

$G_d = m_d G_{d_o}$ where $m_d \in RH^\infty$ is inner and $G_{d_o} \in RH^\infty$ is outer. Under these assumptions there exist $X \in RH^\infty$ and $Y \in H^\infty$ such that

$$XG_n + YG_d = 1. \tag{28}$$

(To construct solutions of (28), X must be chosen to satisfy a set of interpolation constraints at the zeros of G_d in the closed unit disc so that $Y = (1 - XG_n)/G_d$ belongs to H^∞. Since the constraints are finite in number, X can always be chosen to be rational.) The set of all controllers which stabilize the plant can now be written in the form

$$C = \frac{X + QG_d}{Y - QG_n}$$

for some $Q \in H^\infty$. Now let $S := (1 + PC)^{-1}$ and note that

$$S = 1 - XG_n - QG_nG_d. \tag{29}$$

In [73], [74], we show that the computation of

$$\mu = \inf_{\text{stabilizing } C} \left\| \begin{bmatrix} W_1 S \\ W_2(S-1) \end{bmatrix} \right\|$$

where $W_1, W_2 \in RH^\infty$ are given weighting functions with $W_1^{-1}, W_2^{-1} \in RH^\infty$ may be reduced to computing the norm of the operator

$$\mathbf{A} := \begin{bmatrix} \mathbf{P}_{H(m_v)} \left(W_0(\mathbf{S}) - \hat{W}_0(\mathbf{S}) m(\mathbf{S}) \right) \\ G_0(\mathbf{S}) \end{bmatrix}, \tag{30}$$

where $\mathbf{S} : H^2 \to H^2$ denotes the unilateral shift, $H(m_v) := H^2 \ominus m_v H^2$ and $\mathbf{P}_{H(m_v)}$ the orthogonal projection onto $H(m_v)$, for m, m_v inner functions associated to the plant and weighting filters, and where $W_0, \hat{W}_0, G_0$ are rational H^∞ functions computed from the plant and weighting filters. This reduction is true for plants with arbitrary outer parts. (We can do a similar type of reduction for another two block minimization problem in case the outer part of the numerator of the plant is rational. Namely, in this problem we are required to find

$$\mu = \inf_{\text{stabilizing } C} \left\| \begin{bmatrix} W_1 S \\ W_2 CS \end{bmatrix} \right\|,$$

where $W_1, W_2 \in RH^\infty$ are given weighting functions with $W_1^{-1}, W_2^{-1} \in RH^\infty$.)

In [73], [74], we develop an approach to computing the singular values and vectors of operators of the form (30). We remark here that it is easy to compute the essential norm (see [73], [74]) of the operator $\mathbf{A}$, which will be denoted by $\|\mathbf{A}\|_e$. Then in [74], the following result is proven:

Theorem 7 *Let n denote the maximum of the McMillan degrees of the weighting filters W_1 and W_2, and let ℓ denote the number of unstable poles of the plant P. Then the singular vectors and values of $\mathbf{A}$ which are $> \|A\|_e$ may be derived from an explicitly computable system of $3n + 2\ell$ linear equations (the "singular system").*

In [74], the singular system of equations is explicitly written down. Again the number of equations only depends on the McMillan degrees of the weighting filters, and the number of right half plane poles of the plant. The computation of the *maximal* singular value and the associated singular vectors of **A**, then allows us to find the optimal performance μ of our original control problem and the corresponding optimal compensator.

As an example of the above, consider the plant (in continuous time) with a delay and one unstable pole:

$$P(s) = e^{-hs}\frac{\sigma s + 1}{\sigma s - 1}.$$

The optimal unweighted mixed sensitivity performance can be computed to be

$$\mu = e^{h/\sigma}\sqrt{1 + \sqrt{1 - e^{-2h/\sigma}}}.$$

It is interesting to note that as $h \to \infty$, and/or $\sigma \to 0$, the best achievable performance increases exponentially, as expected. In [35], a more complicated (and realistic) delay problem is worked out using the above methods.

Of course, we would now like to extend the above techniques to unstable distributed multivariable plants in the full four block setting. Research is now proceeding along these directions in our new AFOSR contract. Moreover, we are very interested in working out more benchmark examples in addition to the flight control problem [35] and flexible beam problem [70] to which we alluded above. A key part of the implementation will be to have an efficient way of generating sub-optimal finite dimensional compensators for such problems. This important area is also being investigated.

4.2 Young's Operator

A key difficulty involved in reducing the standard problem to the four block problem is the various kinds of factorizations that must be performed. In fact, one of the major advantages involved in the recent state space methods [33] is that these factorizations may be avoided. Of course as mentioned above, one of the disadvantages of these state space methods is that their practical applicability to distributed systems seems to be very difficult. (On an infinite dimensional state space one gets infinite dimensional, i.e., operator valued Riccati equations. See [28, 29].)

We have recently begun developing a technique which would avoid a number of the problems with such factorizations (especially in the multivariable distributed case [75]), as well as allow the utilization of our frequency domain "skew Toeplitz" methods to distributed systems. First of all, recall that via the Youla parametrization, the standard problem may be formulated as finding

$$\inf_{Q \in H^\infty} \|T_1 - T_2 Q T_3\|_\infty,$$

where T_1, T_2, T_3, Q are matrix-valued H^∞ functions of compatible sizes. More precisely, let $\mathcal{E}_i, \mathcal{F}_i$ denote finite dimensional complex Hilbert spaces for $i = 1, 2$. Then we take $T_1 \in H^\infty(\mathcal{E}_1, \mathcal{E}_2)$, $T_2 \in H^\infty(\mathcal{F}_2, \mathcal{E}_2)$, $T_3 \in H^\infty(\mathcal{E}_1, \mathcal{F}_1)$, and the parameter $Q \in H^\infty(\mathcal{F}_1, \mathcal{F}_2)$. (In

general, for two complex separable Hilbert spaces $\mathcal{E}, \mathcal{F}$ we define $H^\infty(\mathcal{E}, \mathcal{F})$ to be the space of all uniformly bounded analytic functions in the open unit disc, whose values are operators from $\mathcal{E}$ to $\mathcal{F}$.) We assume that T_2 is inner, and that T_3 is co-inner.

Define

$$\mathcal{H}_1 := L^2(\mathcal{E}_1) \ominus T_3^* K^2(\mathcal{F}_1),$$
$$\mathcal{H}_2 := L^2(\mathcal{E}_2) \ominus T_2 H^2(\mathcal{F}_2),$$

where $K^2(\mathcal{F}_1)$ is the orthogonal complement of $H^2(\mathcal{F}_1)$ in $L^2(\mathcal{F}_1)$. Define the operator (see [94], [36]) $\Lambda : \mathcal{H}_1 \to \mathcal{H}_2$ by

$$\Lambda f := P_{\mathcal{H}_2} T_1 f$$

where $P_{\mathcal{H}_2}$ denotes orthogonal projection on $\mathcal{H}_2$. Then using the commutant lifting theorem, one may show that

$$\inf_{Q \in H^\infty} \|T_1 - T_2 Q T_3\|_\infty = \|\Lambda\|.$$

We would like to be able to compute the optimal performance and parametrize the optimal compensators directly from the Young operator along the lines of the methods that we followed for the four block problem [44], [45], [75]. To do this would be a great advantage since we would no longer have to go through the messy transformation of the standard problem to the four block form. We are now developing a straightforward way of computing the H^∞-optimal performance and compensators with the Young operator based on a certain triangular representation of the operator.

4.3 Mixed Norm Suboptimal Controllers

One of the main themes in modern control theory is the utilization of norm based criteria to measure the optimal performance of a given control system. (See [32] for detailed discussions on this topic.) Two of the most important norms employed in modern control analysis and design are the H^2-norm and the H^∞-norm. Indeed, the H^2-norm is the basis of classical quadratic optimal control, while the H^∞-norm appears in modern robust synthesis (and implicitly in the more classical loop shaping methods [32]).

We have been investigating combining the advantages of both H^2 and H^∞ control in [38]. (See also [36], [65]) for extensive lists of references on various approaches to mixed norm control.) Our starting point is a nice result of Kaftal-Larson-Weiss [62] which guarantees the existence of an interpolant which simultaneously satisfies an H^∞ and H^2 suboptimality criterion. Using the theory described in [43] on suboptimal H^∞ interpolants, we can then give an explicit way of computing the combined H^∞–H^2 suboptimal interpolant in a given H^∞ problem.

More precisely, for $c \in L^\infty$, set $d_\infty(c) :=$ distance(c, H^∞), and $d_2(c) :=$ distance(c, H^2). (Note that all of our Hardy spaces are defined on the unit disc here.) Then for any $\delta > 1$, it is proven in [62] that there exists $\phi \in H^\infty$ such that

$$\|c - \phi\|_\infty \leq \delta d_\infty(c), \; \|c - \phi\|_2 \leq \frac{\delta d_2(c)}{\sqrt{\delta^2 - 1}}. \tag{31}$$

In case, $m, w \in H^\infty$ with m inner, and $w = p/q$ rational, using skew Toeplitz theory [38] one may explicitly construct ϕ, i.e., if $c := \overline{m}w$, then we can find $\phi \in H^\infty$ satisfying

$$\|\overline{m}w - \phi\|_\infty \leq \delta d_\infty(c), \quad \|\overline{m}w - \phi\|_2 \leq \frac{\delta d_2(c)}{\sqrt{\delta^2 - 1}}. \tag{32}$$

Indeed, let $\rho := \delta d_\infty(c)$, and let

$$\hat{A}_\rho := \rho^2 q(T)^* q(T) - p(T)^* p(T)$$

be a skew Toeplitz operator. Let g_δ denote the function in H^∞ defined by

$$g_\delta := \delta^2 d_\infty(c)^2 \hat{A}_\rho^{-1} q(T)^*(1 - m\overline{m(0)}). \tag{33}$$

Then the function

$$w - m\phi = B_\delta = \frac{p(T)g_\delta}{m\overline{m(0)} + q(T)g_\delta} \tag{34}$$

satisfies (32). Explicitly, $\phi = -\overline{m}(B_\delta - w) \in H^\infty$ and

$$\|w - m\phi\|_\infty \leq \delta d_\infty, \quad \|w - m\phi\|_2 \leq \frac{\delta d_2}{\sqrt{\delta^2 - 1}}.$$

The skew Toeplitz approach is the only procedure of which we are aware to compute a ϕ satisfying (32) when m is irrational. Further, let $n := \max\{\text{degree } p, \text{degree } q\}$. Then, if m is Blaschke product of order k where n is small and k is large, the methodology just outlined provides a numerically efficient method for computing the function ϕ satisfying the Kaftal-Larson-Weiss constraints (32). However, if this is not the case and m is rational, then the state space methods in [36] to compute a ϕ satisfying (32) may be more efficient.

Since as it turns out that the combined H^∞-H^2 interpolant is a central solution, these interpolants are well-known to be numerically robust, and leads to an interesting class of combined H^∞-H^2 controllers for feedback systems. We are now actively investigating the properties of these controllers, as well as extending these results to find a computational procedure for the *full standard problem* setting for distributed MIMO systems.

Further, we have found a new method for constructing the *central solutions* as related to the general H^∞ standard problem in the multivariable case. We are now in the process of exploiting this technique for the explicit parametrization of the suboptimal controllers for multivariable distributed systems. This would give a powerful alternative method to the one-step extension techniques we have been previously using.

Finally, we are interested in combining other types of norms in control, for example, L^1 and H^∞. A possible method for doing this can be based on some new results on interpolation in Banach spaces due to Pisier [78] and Janson [61]. In fact, this methodology can also have some nice applications to model reduction (relative to "best" Hankel norm approximations), which is certainly a direction which we intend to explore.

5 Image Processing and Computer Vision

We have been doing extensive research into image processing and computer vision, and the use of these methodologies in visual tracking. We are very interested in understanding how to use visual information in a feedback loop. In order to give the reader a flavor of this work, we will concentrate on a theory of scale-space that we developed for planar shapes [81, 82]. In this work, we combine the three important theories of scale-spaces, affine invariant descriptors, and curve evolution, in order to define a new affine invariant scale-space for plane curves. This scale-space is obtained from the solution of a novel curve flow, which admits affine invariant solutions. Our other work in vision covers a new theory of shape [67, 68, 69], as well as new methods for affine invariant image processing and analysis [83, 84, 85].

Multi-scale descriptions of signals have been studied for several years already. A possible formalism for this topic comes from the idea of multi-scale filtering which was introduced by Witkin [93], and developed in several different frameworks by a number of researchers in the past decade. The idea of scale-space filtering is very simple and can be formulated as follows: Given an initial signal $\Psi_0(\vec{X}) : \mathbf{R}^n \to \mathbf{R}^m$, the scale-space is obtained by filtering it with a kernel $\mathcal{K}(\vec{X}, t) : \mathbf{R}^n \to \mathbf{R}^m$, where $t \in \mathbf{R}^+$ represents the scale. In other words, the scale-space is given by $\Psi(\vec{X}, t)$ defined as

$$\Psi(\vec{X}, t) := \Omega_{\mathcal{K}(\vec{X},t)}[\Psi_0(\vec{X})], \tag{35}$$

where $\Omega_{\mathcal{K}(\cdot,t)}[\cdot]$ represents the action of the filter $\mathcal{K}(\cdot, t)$. Larger values of t correspond to images at coarser resolutions.

It is important to note that not every kernel can be used in defining a scale-space. Indeed, several conditions need to be imposed on the signal $\Psi(\vec{X}, t)$ (and therefore on the filtering operation (35)). One of the most important is that of *causality*, which states that no "information" is created when moving from fine to coarse scales.

A famous example of a kernel which satisfies the required conditions is the Gaussian kernel. In this case, the scale-space is linear, and the filter in (35) is just defined by convolution. The Gaussian kernel is one of the most studied in the theory of scale-spaces. It has some very interesting properties, one of them being that the signal Ψ obtained from it, is the solution of the heat equation (with Ψ_0 as initial condition) given by

$$\frac{\partial \Psi}{\partial t} = \Delta \Psi.$$

(For more details about the Gaussian scale-space, see the aforementioned references.)

One of the key facts that can be gleaned from the Gaussian example, is that the scale-space can be obtained as the solution of a partial differential equation called an *evolution equation*. This idea was developed in different works for evolution equations different from the classical heat equation. In what follows, we will describe scale-spaces for planar curves which are obtained as solutions of nonlinear evolution equations.

The second fundamental concept which we would like to emphasize is that of *invariant descriptor*. An invariant descriptor is a property of an object, which does not change when the object undergoes certain transformations. More precisely, a quantity Q is called an *invariant*

of a Lie group $\mathcal{G}$ if whenever Q transforms into $\tilde{Q}$ by any transformation $g \in \mathcal{G}$, we obtain $\tilde{Q} = \Theta Q$, where Θ is a function of g alone. If $\Theta = 1$ for all $g \in \mathcal{G}$, Q is called an *absolute invariant.*

For example, the area is a global absolute invariant descriptor of the Euclidean group (rotations and translations) for planar domains. This means that two planar domains related by an Euclidean motion have the same area. The well-known Euclidean curvature given as a function of arc-length is a local absolute invariant of the Euclidean group, and a relative invariant of the similarity group. We should add that the development of invariants of viewing transformations (Euclidean, similarity, affine, and projective maps) has received extended attention from the computer vision community in the last years. The topic is very important in areas such as object recognition.

A special class of invariants is given by the *differential* invariants which are based only on local information, and are useful for the representation and recognition of objects under partial occlusions. An example of this kind of invariants is the Euclidean curvature mentioned above. The theory of differential invariants is classical and goes back at least to the work of Gauss on Euclidean geometry. Much of the recent related work in computer vision is based on those old theories. In our work, we have been especially interested in affine differential invariants.

The third component of our work comes from the theory of plane curve evolution. Here the curve regarded as the boundary of a planar domain, deforms in time. This deformation is governed by a partial, usually nonlinear, evolution equation. Different evolution equations can model different physical phenomena, such as crystal growth, the Huygens principle, and curve shortening processes. The theory has been well studied in areas such as computational physics, differential geometry, numerical analysis, and parabolic equations. We have introduced these ideas into computer vision in [67, 68, 69].

Motivated by the importance of affine transformations in computer vision, a new theory of affine curve evolution was recently developed [81, 82, 83, 84]. This theory, based on affine differential geometry and the theory of parabolic evolution equations, constitutes the basis of the work on multi-scale representations of signals and image processing in general. This is based in turn on a new nonlinear evolution diffusion equation which admits affine invariant solutions. The solution of this evolution equation determines an *affine* invariant scale-space for planar curves. Efficient computer implementation of this theory is possible due to a recently developed numerical algorithm for curve and surface evolution. In our continuing AFOSR research, we will be employing these algorithms to a number of problems in active vision, and visual control.

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7 Papers of Allen Tannenbaum and Collaborators under AFOSR-90-0024

(All the authors are listed in alphabetical order on the following papers.)

1. "Skew Toeplitz approach to the H^∞-control of multivariable distributed systems" (with H. Özbay), *SIAM J. Control and Optimization* **28** (1990), 653-670.

2. "On certain minimal entropy extensions appearing in dilation theory" (with C. Foias and A. Frazho), *Linear Algebra and Its Applications* **137** (1990), 213-238.

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4. "A spectral commutant lifting theorem" (with H. Bercovici and C. Foias), *Transactions of AMS* **325** (1991), 741-763.

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7. "The invariant subspaces of a uniform Jordan operator" (with H. Bercovici), *Journal Math. Analysis and Applications* **156** (1991), 220-230.

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